Printed Pages – 6 Roll No. : ....

B022411(014)

B. Tech. (Fourth Semester) Examination, April-May 2021

(AICTE Scheme)

(CSE Engg. Branch)

DISCRETE MATHEMATICS

Time Allowed : Three hours

Maximum Marks : 100

Minimum Marks : 35

Note : *Part (a) of each question is compulsory and carries 4 marks each. All questions are required to be answered, selecting any two from (b), (c) and (d) and carries 8 marks each.*

1. (a) Construct converse, inverse and contrapositive of the direct statement :

   "If $4x - 2 = 10$ then $x = 3$."

   (b) Test the validity of the argument :

"If 8 is even then 2 does not divide 9. Either 7 is not prime or 2 divides 9. But 7 is prime therefore 8 is odd."

(c) State and prove De-Morgan's law in a Boolean algebra $(B,+,\cdot,')$.

(d) Change the following boolean function to disjunctive normal form and conjuctive normal form :

$$f(x,y,z)=(x+y+z)(xy+x'z)'$$

2. (a) Let $A=\{1,2,4\}$, $B=\{2,5,7\}$ and $C=\{1,3,7\}$, then show that

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

(b) If I is the set of integers and the relation

'$xRy \Leftrightarrow x-y$ is an even integer'.

then prove that R is an equivalence relation, where $x,y\in I$.

(c) Show that the inclusion relation '$\subseteq$' is a partial

ordering on the power set $P(s)$ and draw the Hasse diagram for the partial ordering $\{(A,B)\mid A\subseteq B\}$.

(d) If $f:X\to Y$ be a one-one and auto mapping then prove that

$$f\circ f^{-1}=I_y \text{ and } f^{-1}\circ f=I_x$$

where I_x and I_y are identity mappings of X and Y respectively.

3. (a) Prove that every cyclic grup is an abelian group.

(b) Let Q_+ be the set of all positive rational numbers and * is a binary operation on Q_+ defined as

$$a*b=\frac{ab}{3}, \forall\, a,b\in Q_+$$

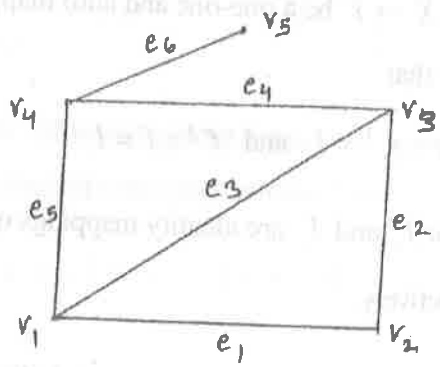
show that $(Q_+,*)$ is a group.

(c) State and prove lagrange's theorem.

 PTO

(d) Prove that every field is an integral domain.

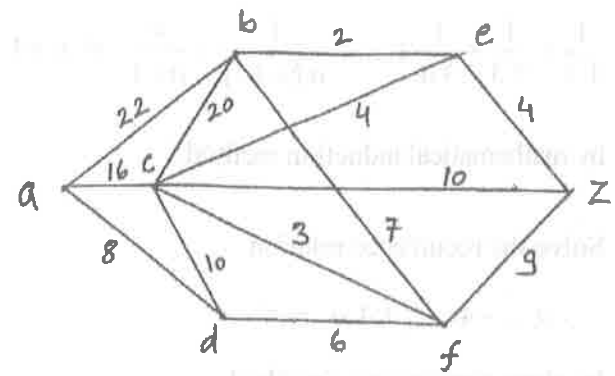
4. (a) Define incidence matrix and find the incidence matrix of the following graph :



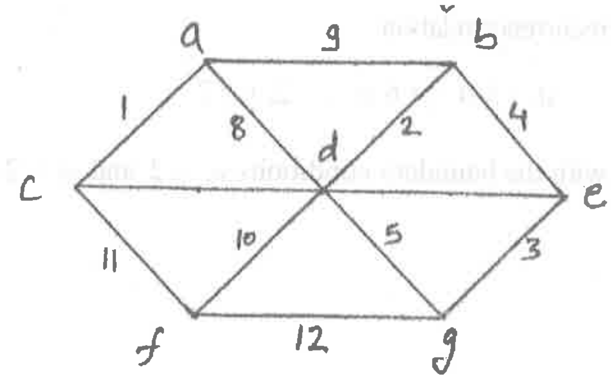
(b) (i) Prove that the sum of the degrees of odd vertices in a graph is always an even number.

(ii) Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(c) Find the shortest path between a and z for the graph given below, where the numbers associated with the edge are the distances between vertices :



(d) Find a minimal spanning tree of the graph :



5. (a) State generalized pigeonhole principle. Show that if seven colours are used to paint 50 cars, atleast eight cars will have the same colour.

(b) Show that :

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n.(n+1)}=\frac{n}{n+1}, \forall\ n\geq 1$$

by mathematical induction method.

(c) Solve the recurrence relation

$$a_{r+2}-4\ a_{r+1}+4\ a_r\ = r^2$$

by characteristics root method.

(d) Solve by the method of generating functions, the recurrence relation

$$a_r-5\ a_{r-1}+6\ a_{r-2}=2,\ r\geq 2$$

with the boundary conditions $a_0=2$ and $a_1=2$.